Performance of Factor Models in a Simple Economy

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Overview

What We Do

- Simulate the Berk-Green-Naik (1999) model
  - 1,000 firms, monthly data, 60 years
  - Calculate return, size, book-to-market, ROE, asset growth, and momentum
  - Repeat 300 times
- Evaluate factor models defined from characteristics
  - Fama-French
  - Fama-MacBeth-Rosenberg
  - Didisheim, Ke, Kelly, and Malamud (2024) complexity
  - Kelly, Pruitt, Su (2019) instrumented PCA

- Assess models by Hansen-Jagannathan distances and Sharpe ratios
- Can calculate true conditional moments and true conditional SDF
- Conclusions:
    - KPS $>$ DKKM $>$ Fama-French and Fama-MacBeth-Rosenberg
    - Performance improves in DKKM up to hundreds of factors
    - Suggests hybrid model (next on our agenda)

Berk-Green-Naik Model

Projects

- One-period SDFs are iid lognormal.
- Vasicek interest process. Shocks correlated with SDF.
- Firms start with zero capital. Get one project per month. Dies if not taken.
- Each invested project $\rightarrow$ operating cash flows monthly until random death. Cash flows correlated with SDF.
- Each project has unique SDF beta and unique idiosyncratic risk.
- Take all positive NPV projects (depends on beta and interest rate).
- No debt. Free cash flow paid out to shareholders.

Firm Values and Characteristics

- Value of growth options same for all firms (depends on interest rate).
- Assets in place depends on past project characteristics and past interest rates..
- Value of assets in place also depends on current interest rate.
- Can calculate size, book-to-market, profitability, asset growth, and momentum.

Mechanisms

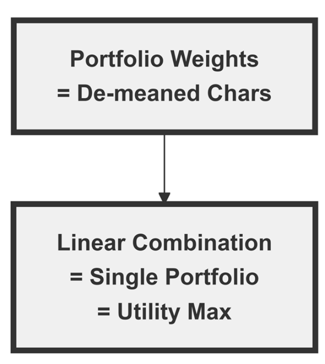
- Small firms are mostly growth options. Riskier and higher expected return.
- Given market cap, value firms have more projects (higher book) of lower average value. Low project value $\sim$ high beta $\sim$ high expected return.

Panels

- Simulate 300 panels of 1,000 firms and 920 months
- Discard first 200 months to reach steady state
- Calculate unique SDF in span of assets each month in each panel

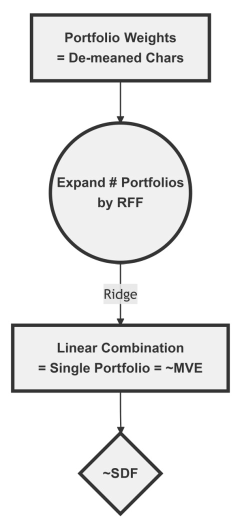
Factor Models

Background: Brandt/Santa-Clara/Valkanov (2009)



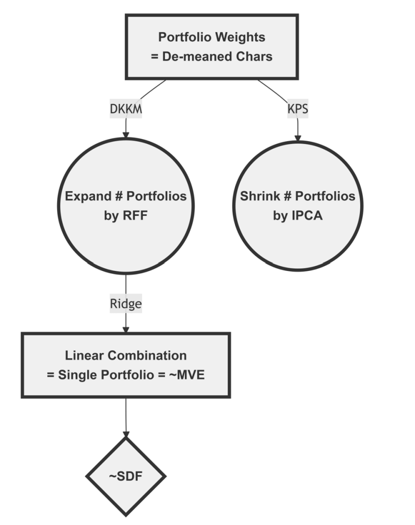
- Start with one portfolio for each characteristic.
- For high-minus-low portfolio, go longer when char is higher, shorter when char is lower.
- In other words, portfolio weights are standardized (monotone in) characteristic values.
- Choose the linear combination (portfolio of portfolios) that achieves the highest average utility in past sample.

Didisheim, Ke, Kelly, and Malamud (2024)



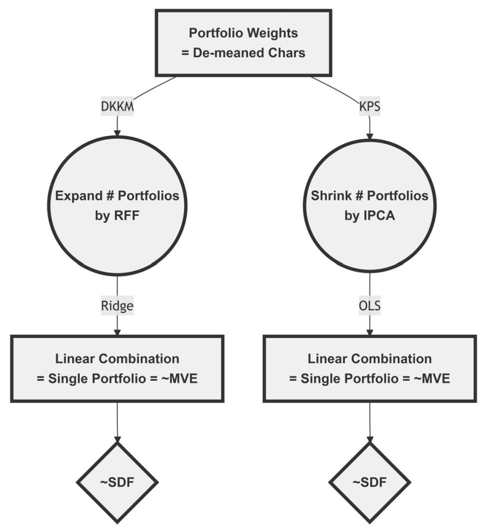
- Expand number of characteristics (= portfolios) to as many as 1 million
  - Random linear combinations
  - Then sines and cosines
- Ridge $\Leftrightarrow$ maximize quadratic utility with penalization (Hansen/Richard, Britten-Jones)
- Approximate frontier portfolio generates approximate SDF

Kelly, Pruitt, and Su (2019)



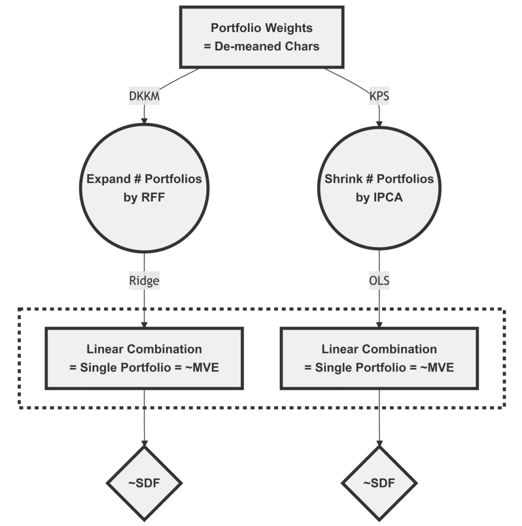
- Latent factor pricing model (small # of factors)
- Factor loadings are linear combinations of characteristics plus noise.
- Estimate latent factors

Kelly, Pruitt, and Su SDF



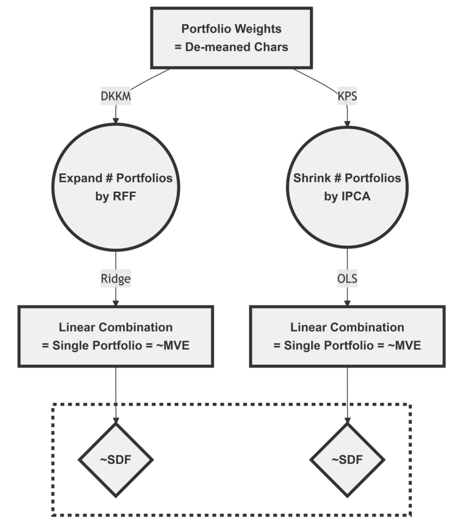
- Can apply ridge (or OLS) as in DKKM to estimate SDF
- OLS probably better with small # of factors

Sharpe Ratios



- Calculate conditional Sharpe ratios each month
- Average across months in each panel
- Produces 300 observations for each model

Hansen-Jagannathan Distances



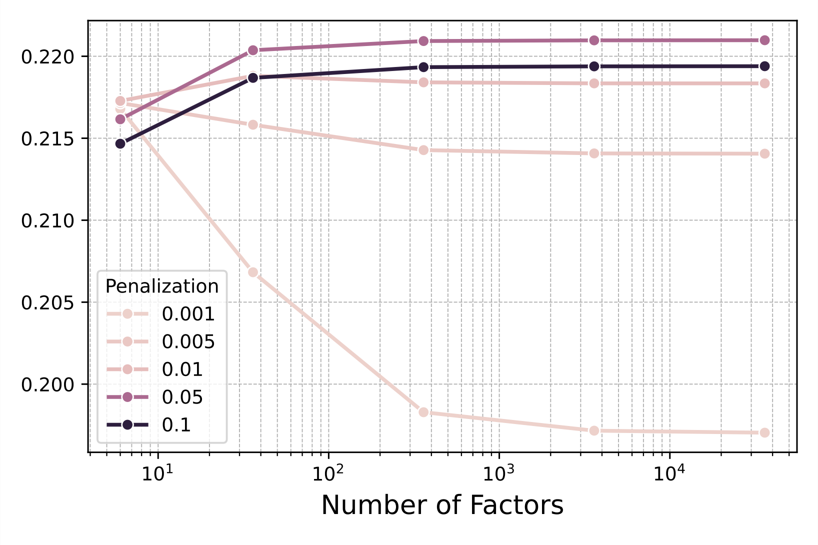
- Calculate realization of $(\widehat{\text{SDF}} - \text{SDF})^2$ each month
- Average across months in each panel
- Produces 300 estimates of unconditional HJ distance for each model

Classical Models

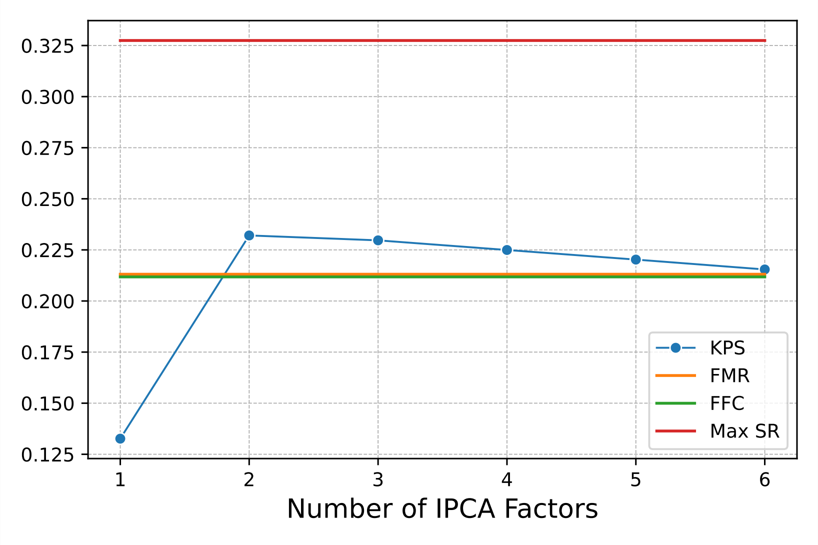
- Fama-French-Carhart 6-factor model
- Fama-MacBeth-Rosenberg 6-factor model
    - Fama-MacBeth regression coefficients are linear functions of returns, hence portfolio returns.
    - Weights in implicit portfolios sum to zero, hence long-short portfolio returns.
    - Scale weights to sum to 1 on both long and short sides.
- Get single portfolio and SDF as for KPS (use OLS).

Results

Performance of DKKM (Sharpe Ratios)



KPS, FFC, & FMR (Sharpe Ratios)



FMR vs FFC

	Sharpe Ratio	HJ Distance
FMR	0.213	0.239
FFC	0.212	0.238
FMR - FFC	0.001	0.001
t-stat	1.779	0.499
p-value	0.076	0.618

t-statistics for DKKM vs FMR (Sharpe Ratios)

# factors	6	36	360	3600	36000
penalization					
0	3.61	-28.80	-117.87	-112.57	-112.54
0.001	3.78	-6.32	-14.26	-15.26	-15.37
0.005	4.07	2.90	1.24	1.02	1.01
0.01	4.12	6.13	5.69	5.61	5.61
0.05	2.77	7.34	8.02	8.08	8.09
0.1	1.34	5.30	6.04	6.11	6.12

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t-statistics for DKKM vs FMR (HJ Distances)

# factors	6	36	360	3600	36000
penalization					
0	-6.89	13.54	38.74	44.47	45.99
0.001	-6.91	3.72	8.12	8.63	8.76
0.005	-6.41	-4.87	-3.19	-3.05	-3.03
0.01	-5.61	-9.01	-8.41	-8.38	-8.36
0.05	-0.87	-5.66	-6.52	-6.63	-6.64
0.1	2.28	-0.99	-1.62	-1.69	-1.70

t-statistics for KPS

Sharpe Ratios

# factors	1	2	3	4	5	6
vs DKKM	-47.18	23.11	13.57	5.49	-0.84	-6.34
vs FMR	-39.09	19.26	22.01	19.00	12.63	4.67

Hansen-Jagannathan Distances

# factors	1	2	3	4	5	6
vs DKKM	26.92	-3.42	-2.05	0.34	3.20	5.02
vs FMR	11.46	-12.52	-12.65	-9.60	-4.44	-0.26

Conclusion

Conclusion

- DKKM outperforms FFC and FMR in the BGN model.
  - Performance is increasing in number of factors up to several hundred, despite simplicity of the BGN model.
  - Performance plateaus at several hundred factors but does not deteriorate up to 36,000 factors.
  - Dispersion of panel statistics is less in DKKM model than in FFC and FMR, despite the fact that factors are randomly generated.
- KPS outperforms DKKM.
- Possible hybrid model: replace ridge in DKKM with IPCA and OLS.