

Do Common Factors Really Explain the Cross-Section of Stock Returns?

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Discussion by

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2022 Utah Winter Finance Conference

Overview

Use random forest to estimate conditional expected returns

SVD and 'Fama-MacBeth' → PC factors (50, 100, ...) and hedged portfolio

Hedged portfolio has Sharpe ratio $\approx 150\%$ and $>$ Sharpe ratio of unhedged portfolio

If hedged portfolio is truly hedged against factor risk, this seems to violate rational pricing (APT).

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Outline of Discussion

- 1 Simple example of hedge portfolio construction to illustrate effect of estimation errors
- 2 Review of paper's methodology
- 3 Results of a simulation
- 4 Other comments

Simple Example

CAPM economy with three assets. Monthly returns from 1926.
Three assets are:

- 1 Mkt-RF
- 2 Residual from regression of SMB of Mkt-RF and constant
- 3 Residual from regression of HML on Mkt-RF and constant

True betas = 1, 0, 0

True means = 0.691, 0, 0 (0.69 = 69 bp per month)

Add beta estimation error = ± 0.1 and mean estimation error
= ± 0.001 .

Hedged Portfolio Example

Hedged portfolio at date t defined by $w'v = 0$, $w'\beta = 0$, and $w'm = 1$, where β = estimated conditional betas and m = estimated conditional means.

Usually minimize $w'w$ but 3 assets and 3 constraints here

Rescale w so that long and short sides sum to 1

Example:

$$\beta = \begin{pmatrix} 0.9 \\ 0.1 \\ -0.1 \end{pmatrix}, m = \begin{pmatrix} 0.692 \\ -0.001 \\ -0.001 \end{pmatrix} \Rightarrow w = \begin{pmatrix} 0.2 \\ -1.0 \\ 0.8 \end{pmatrix}$$

Annualized Sharpe ratio is 11.7%.

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Errors in Variables Bias

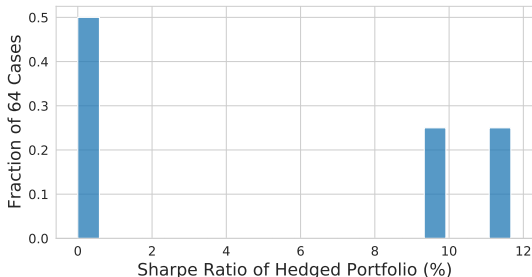
Except for the rescaling, the return of the hedged portfolio w is the FM regression coefficient for the characteristic m in a regression on β and m (Rosenberg, 1974; Fama, 1976).

Because betas are estimated with error and m is positively correlated with true betas, the FM coefficient has a positive mean.

The positive mean/Sharpe ratio of the hedged portfolio is the OLS errors-in-variables bias from using estimated betas on the right-hand side of the FM regression.

Sharpe Ratio Bias

In this example, the \pm error for each of 3 betas and each of 3 means gives $2^6 = 64$ different combinations. The Sharpe ratios of the 64 hedged portfolios are summarized in the following.

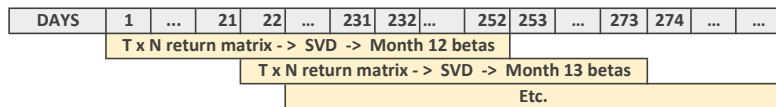


Sharpe ratio is never negative and is zero only when errors of β_2 and β_3 are same. The beta estimates are unbiased across these 'samples,' but the Sharpe ratio has an EIV bias.

Singular Value Decomposition Betas

One-year panels, rolling one month at a time. Return matrix of size $252 \times$ number of stocks ($T < N$).

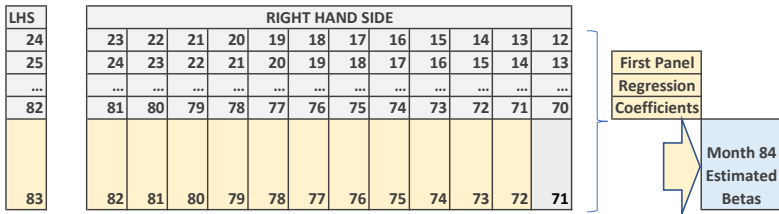
SVD $\Rightarrow T = 252$ eigenvectors (beta vectors) of dimension N .



Month 12 betas calculated at end of month 12, etc. Standard thing would be to use past 252 days to estimate next month's betas (e.g., first 252 days \rightarrow month 13 betas instead of month 12 betas), but here there is another step.

Forecasting Betas

Regress monthly SVD betas on 12 lags in panels of 60 months.
Use coefficients and 12 betas to forecast next month betas.



Factors and Portfolios

Factor h min $w'w$ subject to $w'\iota = 0$, $w'\beta_j = 0$ for $j \neq h$ and $w'\beta_h = 1$. Construct first 50, 100, ... factors

Unhedged min $w'w$ subject to $w'\iota = 0$ and $w'm = 1$.

Hedged min $w'w$ subject to $w'\iota = 0$, $w'\beta_h = 0$ for first 50, 100, ... eigenvectors, and $w'm = 1$

These are Fama-MacBeth portfolios (Fama 1976), but rescale so both long and short sides sum to 1.

Also called BARRA factors (Rosenberg 1974).

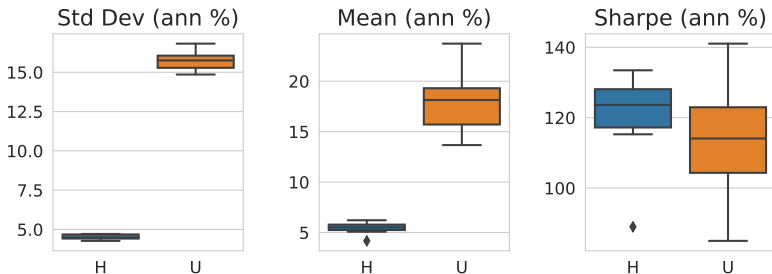
Simulation

- 1 6-factor model for daily returns calibrated to FF-Carhart
- 2 random betas calibrated to cross-sectional distribution, independent across stocks and constant over time
- 3 50 years of daily returns with independent homoscedastic idiosyncratic shocks
- 4 use SVD as in paper on 3000×252 demeaned return matrix to obtain betas at end of each month
- 5 Forecast betas using rolling 60-month panel regressions on 12 lags as in paper
- 6 Construct factors, hedged, and unhedged portfolios as in paper, but using true stock means instead of random forest predictions
- 7 Repeat 10 times

Sharpe Ratios

Hedged portfolio has lower risk and lower mean than unhedged portfolio but usually higher Sharpe ratio.

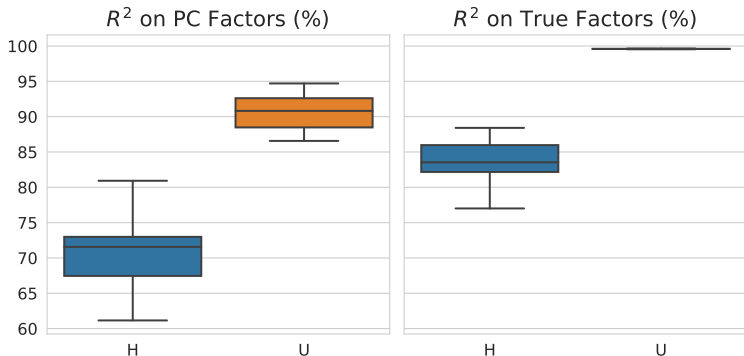
Figure shows distribution across simulations (H=hedged, U=unhedged).



Hedged Portfolio has Factor Exposure

Hedge portfolio loads on PC factors due to beta estimation errors.

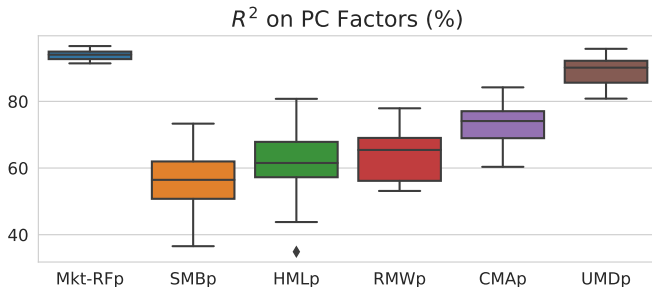
It loads even more on the true factors (PC factors don't span true factors – next slide)



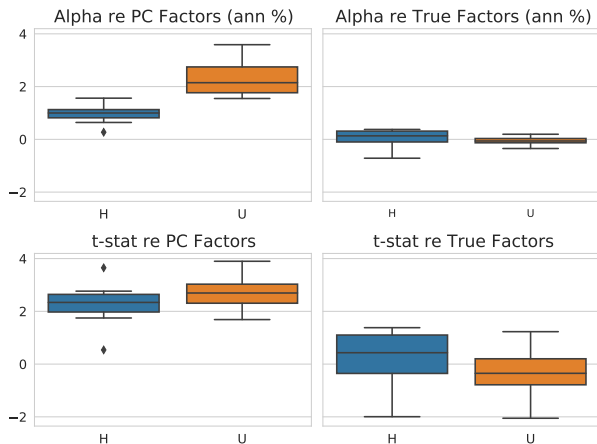
PC Factors Don't Span True Factors

The simulated economy is a six-factor APT economy. The factor names don't really matter (there is no size, book-to-market, etc. in the simulation).

The true factors (projections) are only partially spanned by the PC factors.



Portfolios Have Alpha re PC Factors



Also, GRS with true factors as test assets rejects the PC factor model in 9 of the 10 simulations at 10%.

Other Comments

- 1 Eigenvalues are biased upwards and inconsistent in large N setting ($N/T \rightarrow c > 0$). So, PC factors may explain less of the variation in the data than it appears.
 - Harding, Explaining the single factor bias of arbitrage pricing models in finite samples, *Economics Letters*, 2008.
 - Allen and Perry, Singular value decomposition and high dimensional data, *Encyclopedia of Econometrics*, 2013.
- 2 Low risk premia of PC factors is somewhat surprising. Sharpe ratios of first five PC factors are lower in the data than in the simulation. Would be useful to plot max squared Sharpe ratio of first n factors for $n = 1, \dots, 50$. Are there really no risk premia in the factors?
- 3 Also, report alpha and R^2 of hedged portfolio on PC factors. How much of the performance comes from PC factor risk?